Perception, 2005, volume 34, pages 121 – 123

DOI:10.1068/p5359

Last but not least

A sculpture technique for rendering complex impossible objects

Artists have drawn many types of three-dimensional objects that look like realistic solids but are physically impossible. One interesting class of impossible depictions consists of objects with reasonable parts that are conjoined into impossible wholes. This class includes Reutersvard's and Penrose's impossible triangles, and Escher's staircases. In general, the joints are spaced apart, and the impossibility of the object is revealed only by taking a visual walk around the object, similar to the visual walk required to see that a Mobius strip is one-sided. Drawings of impossible objects look strange only if incongruent joints are close enough to be attended to simultaneously (Hochberg 1968). Various attempts have been made to create three-dimensional sculptures that appear to be impossible objects (Gregory 1968). Many such sculptures can be explored at the website http://www.sandlotscience.com/Impossible/impos_frm.htm. The trick in these sculptures depends on their appearing to be closed objects when seen from a particular view. Small changes from this viewpoint reveal the hidden gap, and expose the solid as a non-impossible object. In addition, these sculptures are quite limited in their complexity as compared to many impossible drawings. For the class of rectangular polyhedra, ie those composed of mutually perpendicular faces, Sugihara (1977) has derived a mathematical procedure to decide which anomalous pictures can arise from realizable solids. Sugihara also describes a system for generating unfolded surfaces of the realizable polyhedra. Once the surfaces are folded, the resulting polyhedra are not rectangular, but appear to be so from the accidental viewpoint corresponding to the anomalous pictures.

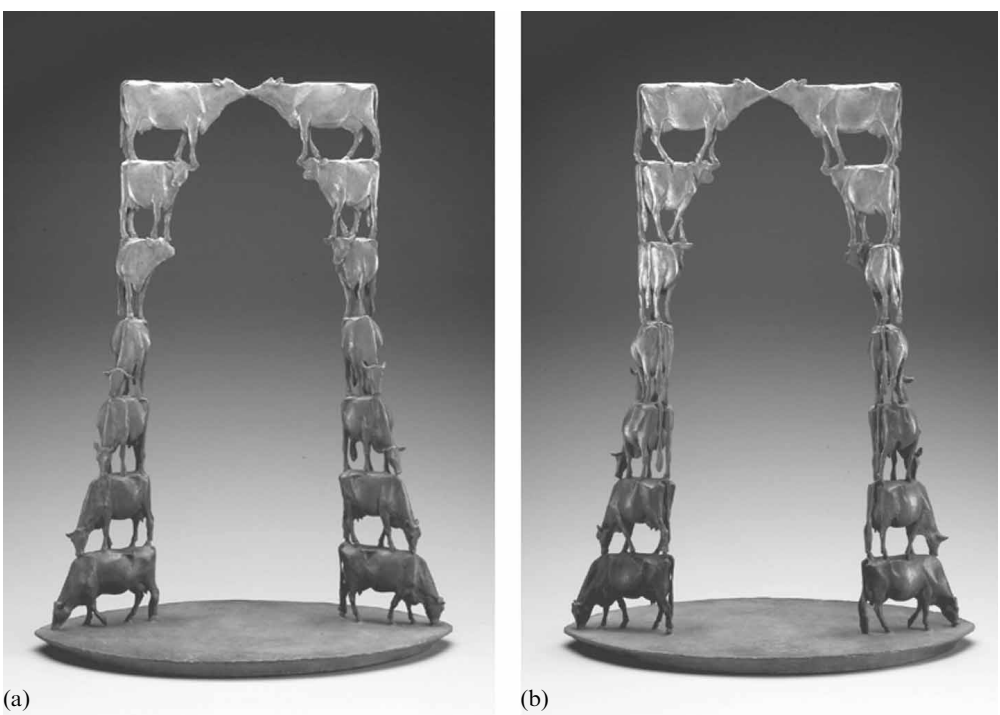

Figure 1. Front view (left) and back view (right) of *Segue* by Anne Huibregtse. Overall dimensions of *Segue*: height 18.25 inches; length, side to side 13.75 inches; depth, front to back 7.5 inches.

Recently, I chanced upon the bronze sculpture *Segue* by Anne Huibregtse. Figure 1 (left) shows the front view of the sculpture. The observer perceives three-dimensional cows standing on top of each other, forming an arch. Figure 1 (right) shows the back view of the sculpture. It is not usual for cows to stand on each other, but aside from this whimsy, the sculpture looks realistic. Closer perusal, however, reveals that this is an impossible object. The cows are standing obliquely on top of each other. If a cow stands straight, at an angle to the cow beneath it, at least one pair of hooves must stand on air. Yet, here, all four hooves of each upper cow rest on the back of each lower cow. This sculpture thus fits the definition of an impossible object as composed of reasonable parts conjoined in impossible ways. There is an overall impossibility here as well. If each cow is standing stably on top of another, they should form columns, not an arch that spans more than two cow-lengths. This sculpture is so convincing, that it takes lengthy discussion to persuade some observers of the validity of the points above. I have found the following strategy useful: first, think of each cow as cast separately; second, think of placing them obliquely on top of each other to recreate the arrangement in the sculpture. Each cow must turn through 30° if we are to rotate the cows through 180° with five intervening cows. The top of a cow's back is quite narrow and it is not possible to get one cow to stand at 30° on the back of another. Suppose a cow's back is 6 feet long, then the front feet would have to be about 3 feet off the midline in order to get the necessary twist (6 times $\sin 30^\circ$). It is thus clear that a stable arch is a physical impossibility.

Figure 2 is a side view that reveals the secret that makes these illusions possible. The sculpture is not completely in the round; instead the 'squashed' cows are much narrower than they seem from the front and back views. They are actually carved in relief to give the illusion of roundness. The cows also line up on top of each other, instead of being placed obliquely. Unlike most relief sculptures, *Segue* is double-sided.

Figure 2. Side view of *Segue.* Dimensions of cows (bottom-to-top, length × width):

$3\frac{5}{8}$ inches × $1\frac{1}{16}$ inches and $3\frac{3}{4} \times 1\frac{1}{16}$ inches
$2\frac{7}{8}$ inches × $\frac{15}{16}$ inch and $2\frac{3}{4}$ inches × $\frac{29}{32}$ inch
$2\frac{1}{8}$ inches × $\frac{3}{4}$ inch and $1\frac{7}{8}$ inches × $\frac{13}{16}$ inch
$1\frac{7}{16}$ inches × $\frac{5}{8}$ inch and $1\frac{1}{2}$ inches × $\frac{5}{8}$ inch
$1\frac{3}{4}$ inches × $\frac{9}{16}$ inch and $1\frac{7}{8}$ inches × $\frac{15}{32}$ inch
$2\frac{11}{16}$ inches × $\frac{7}{16}$ inch and $2\frac{1}{2}$ inches × $\frac{7}{16}$ inch
$4\frac{1}{8}$ inches × $\frac{3}{8}$ inch and $4\frac{1}{8}$ inches × $\frac{3}{8}$ inch

The bottom cows were measured head-to-tail and side-to-side. By the third tier, length is back leg to ear, and width is from belly to shoulder or rump. For middle tiers, length is belly side-to-side, and width is head-to-tail. These measurements show how the cows are squashed and get progressively narrower as they go up.

It is not carved against a background, but with spaces between forms. The technique of making such a sculpture is not simple. It requires translating into three dimensions the perspective tricks that are used in drawing or painting on a flat surface. These tricks create the impression that the cows are slanted with respect to each other.

The perceived volume of each cow is a result of shape-from-shading. Closer perusal reveals that the patterns of light and dark on the cows are consistent with the presence of one light source within each local curvature, but the lighting cues on the whole sculpture are not consistent with a single light source. Again, it seems that the visual system is willing to settle for illuminant consistency within parts and is not thrown off by impossible illuminant conjunctions across parts. On the basis of much more simplistic stimuli, Ramachandran (1988) claimed that the brain uses a tacit assumption that there is only a single light source illuminating the image, and prefers a 'common-light-source' assumption to a 'common-depth' assumption. The images of this sculpture show that these assertions are wrong for even moderately complex situations.

The rewards of Huibregtse's technique are that observers are convinced of the reality of an impossible solid object. The technique provides most of the flexibility of drawing impossible objects, but, because the sculpture is itself a solid, the illusion is much more convincing than a drawing or painting of a solid. Its illusions also remain robust under a wide range of viewpoints, unlike the sculptures of impossible triangles or staircases that rely on hidden gaps. The strength of the illusion suggests that, in rendering complex impossible objects, double-sided relief sculpture is a technique worth exploring.

Acknowledgments. Thanks to Anne Huibregtse for providing the photographs, and to Molly Aitken, Hal Sedgwick, Peter Thompson, Fuzz Griffiths, and Andrea Li for suggestions and comments. Photo credits belong to Jim Huibregtse Studio. Preparation of this paper was partially supported by NEI-EY13312.

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ISSN 0301-0066 (print) ISSN 1468-4233 (electronic)

PERCEPTION
VOLUME 34 2005

www.perceptionweb.com